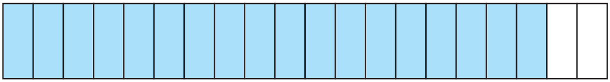
What are percentages?

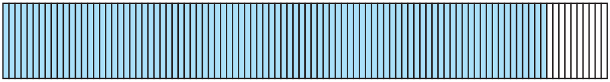
Percentages are a way of using place value to compare between two or more scores or quantities. For example, test scores are often written as a percentage, as is the amount of time that a child may be absent from school.

When a score is written as a percentage, it is converted to being 'out of 100'.

This picture shows a score of 18 out of 20:



Converting it to a percentage can be thought of as turning it into a figure out of 100 like this:



90 of these 100 sections are shaded so we say that 90% is shaded.

The symbol "%" is used to show that a quantity has been converted into a percentage – that is, a number out of 100.

Writing a quantity as a percentage

How to work it out?

The method to write a score as a percentage is often described something like this:

In a test Sarah scored 26 out of 40 marks. To change a score of 26 out of 40 to a percentage first calculate 26 ÷ 40 which gives 0.65.

Now multiply 0.65 by 100 to make a percentage, giving 65%.

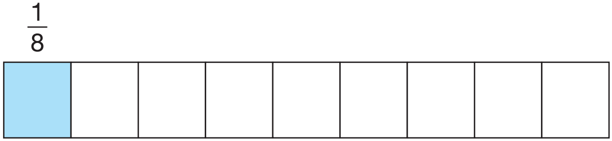
So 26 out of 40 is 65%.

A second example might be to change the fraction $\frac{1}{8}$ to a percentage.
First calculate 1 ÷ 8 which gives 0.125.
Now multiply by 100 to give 12.5%.

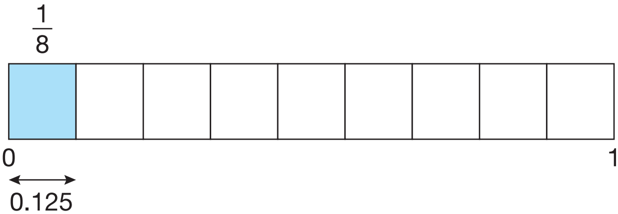
So $\frac{1}{8}$ can be written as 12.5%.

Why does it work?

This rod shows that one out of the eight sections is shaded. This means that the fraction $\frac{1}{8}$ is shaded.

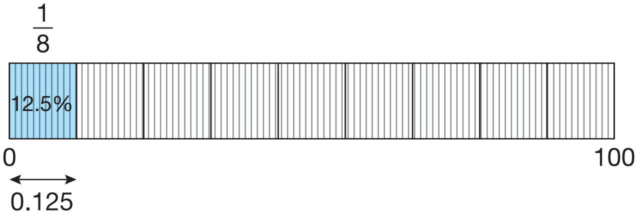


Taking the same rod, with the same proportion shaded and labelling the end as 1:



Then, dividing the 1 into 8 equal sections, or finding $\frac{1}{8}$ of 1, means working out the calculation 1 ÷ 8, which gives the length of each section as 0.125.

Still using the same rod, with the same section shaded, but this time scaling it up to be out of 100 means that the shaded area is also 100 times greater. Using place value we can calculate that 100 times greater than 0.125 is 12.5.



Here there are three different ways to describe the same image. We can say that $\frac{1}{8}$, 0.125 or 12.5% of the rod is shaded.

The fraction, decimal and percentage are just different ways to describe how much of the bar is shaded.

WATCH OUT

When moving from decimal to percentage a calculator display can sometimes cause confusion. For example, the decimal 0.3 is 30% and the decimal 0.03 is 3%. A common mistake is to write a decimal 0.3 as 3%. It is important to think about place value (see chapter 2) to avoid making this error.

TRY IT OUT

Write these decimals as percentages:

0.3	0.335	0.08
0.03	0.95	0.80
0.035	0.59	

NOTE

A percentage can be greater than 100.
For example, the decimal 1.35 can be written as 135%.
136% would be 1.36 and 108% would be written as 1.08.

Write these decimals as percentages:

1.3	1.335	1.08
1.03	1.95	1.80
1.035	1.59	

Write these scores as percentages:

15 out of 20	19 out of 20	1 out of 20
15 out of 40	19 out of 40	1 out of 40
15 out of 80	19 out of 80	1 out of 80
15 out of 100	19 out of 100	1 out of 100

In her science test Matilda scored 18 out of 20 marks and in her maths test she scored 22 out of 25 marks. In which test did Matilda score the higher percentage?

How hard can it be?

To make a percentages question difficult the total may be 'hidden'. For example, if in a class there are 17 girls and 15 boys, to calculate the percentage of the class that are boys, you will first need to find the total number of people. 17 + 15 means that there are 32 children in the class and 15 of them are boys.

If you can follow the reasoning and work out an answer of around 47% then you can feel confident that you can change a fraction into a percentage.

Finding a percentage of an amount

How to work it out

Finding a percentage of an amount is the flip side of writing a score as a percentage.

For example, if there was a test and Sarah scored 65% of the 20 possible marks, how many marks did she score?

To solve this we need to find 65% of 20. In mathematics 'of' can often mean that multiplication is needed.

It is common to hear something like "to find a percentage of an amount you need to multiply the amount by the percentage as a decimal".

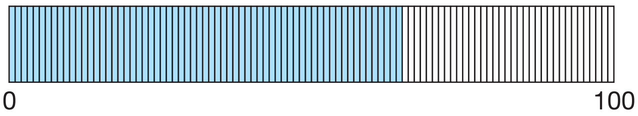
In this case, that means writing 65% as the equivalent decimal (0.65) and then multiplying this by 20:

$0.65 \times 20 = 13$

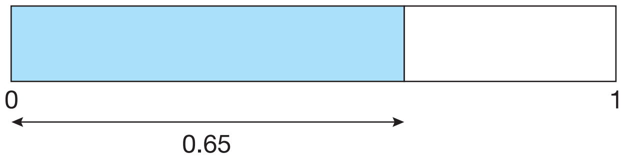
Therefore, you know that 65% of 20 is 13.

Why does it work?

This picture shows a rod split into 100 sections. 65 of them are shaded so 65% of the bar is shaded.



The same percentage of this bar is shaded but this time the bar is 1 unit long. Using place value we know that 0.65 of the bar is shaded:



This rod is 20 times longer than the last one, but the shaded section is still the same percentage. This means that the shaded section must also be 20 times longer:

0.65 × 20 = 13, so 65% of 20 is 13.

The first step converts the percentage to a decimal and the second step scales that decimal so that it is out of the correct amount.

WATCH OUT

As always when working with percentages it is important to take care when converting to decimals. Use what you know about place value to divide and multiply by 100 accurately.

TRY IT OUT

Find…

15% of 38	15% of 98	115% of 38
20% of 38	20% of 98	120% of 38
5% of 38	5% of 98	105% of 38
95% of 38	95% of 98	195% of 38
1% of 38	1% of 98	101% of 38

How hard can it be?

Some percentages are more awkward to convert to decimals than others and it is worth making sure that you are completely confident with this.

If you can use a calculator to work out 10.35% of 32 using just one multiplication then you can be confident that you can find a percentage of an amount.

It is also worth remembering that, as in the above practice, percentages can be greater than 100 so sometimes finding a percentage can increase the value!

Increasing and decreasing by a percentage

How to work it out

When *decreasing* an amount by a percentage, for example, decreasing £70 by 15%, the method can be described as follows:

We know that 100% of the amount is £70.

Decreasing 100% by 15% means that we need to find 85%.

85% of £70 is found by calculating $0.85 \times 70 = 59.5$.

So decreasing £70 by 15% gives £59.50.

When *increasing* an amount by a percentage, for example, increasing £70 by 15% the method can be described as follows:

We know that 100% of the amount is £70.
Increasing 100% by 15% means that we need to find 115%.
115% of £70 is found by calculating $1.15 \times 70 =$ £80.5.

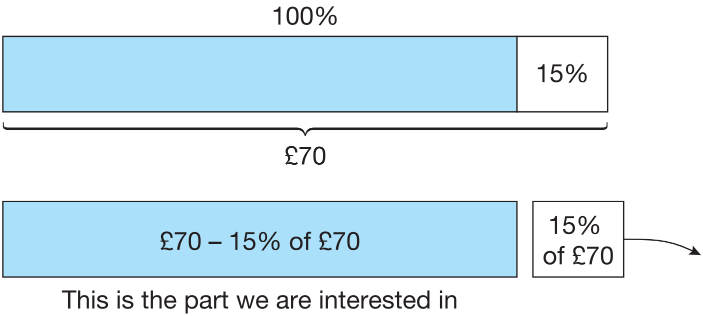
So increasing £70 by 15% gives £80.50.

Why does it work?

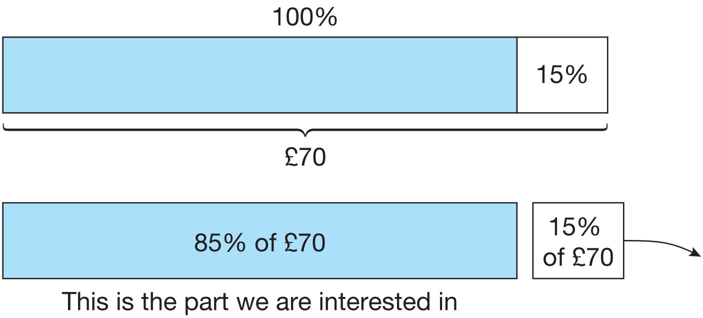
It can be helpful to have a picture in your mind to help understand what is going on.

For example, when decreasing £70 by 15% you might imagine a rod, as in the earlier sections on percentage.

To reduce by 15% one strategy would be to find 15% and take it away, calculating what is left.



However, simplifying this down to one calculation allows for a more efficient and flexible approach.



WATCH OUT

Adding and (particularly!) subtracting decimals can lead to mistakes. When calculating the decimal to use for percentage decreases make sure that you are careful.

You might also occasionally find yourself working out 30% of an amount rather than reducing it by 30% so make sure that you read the question very carefully!

TRY IT OUT

Increase the following amounts by the given percentage

£60 by 15%	£65 by 15%	£165 by 15%
£60 by 25%	£65 by 25%	£165 by 25%
£60 by 35%	£65 by 35%	£165 by 35%
£60 by 5%	£65 by 5%	£165 by 5%
£60 by 95%	£65 by 95%	£165 by 95%
£60 by 0.5%	£65 by 0.5%	£165 by 0.5%

Decrease the following amounts by the given percentage

£60 by 15%	£65 by 15%	£165 by 15%
£60 by 25%	£65 by 25%	£165 by 25%
£60 by 35%	£65 by 35%	£165 by 35%
£60 by 5%	£65 by 5%	£165 by 5%
£60 by 95%	£65 by 95%	£165 by 95%
£60 by 0.5%	£65 by 0.5%	£165 by 0.5%

How hard can it be?

As always with percentages, converting between decimals and percentages is key, and some values can seem more difficult to work with than others. If you can increase and decrease £314 by 3.05% and feel confident in your answers then you can be confident that you are able to calculate percentage changes.

Undoing a percentage increase or decrease

How to work it out

This type of question undoes a multiplication and so it is often stressed that reversing a percentage change means doing a division.

A typical question is something like this:

A shirt is reduced by 15% to £29.75. How much was the shirt originally?

This can then be explained like this:

A reduction of 15% means that 85% is left. Therefore, 85% is equal to £29.75. So, 1% will equal £29.75 divided by 85. That is 35p. The purpose of this is to find the original cost. To find 100% you multiply 0.35 by 100. The answer is £35.

And, in the same way, to calculate the original value after a percentage increase an example is:

If a TV costs £300 including 20% tax, how much would it cost before the tax is added on?

This can then be explained like this:

An additional 20% is written as 1.2 as a decimal.

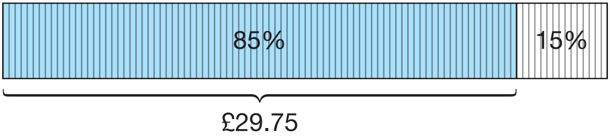
This means that the original price has been multiplied by 1.2 so, to undo that
$300 \div 1.2 = 250$.

So the TV originally cost £250.

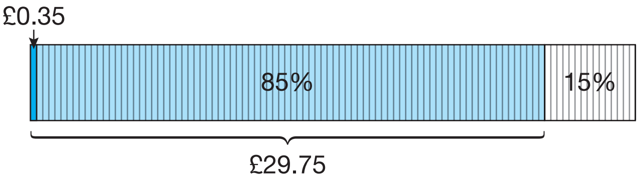
Why does it work?

Let us look at the two examples and use a rod to represent what we know…

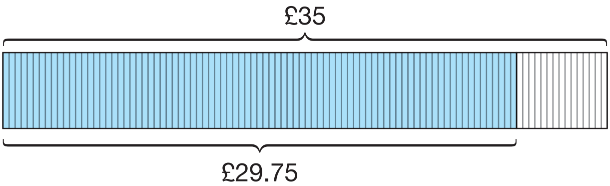
To work out the percentage reduction we know that 85% of the total rod is £29.75:



Dividing by 85 gives the value of 1%:



Then multiplying by 100 gives 100%, the original price.

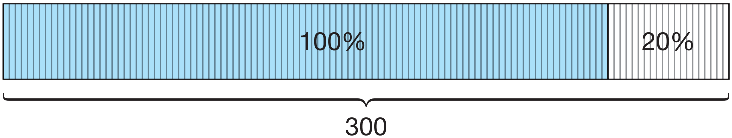


So the original price was £35.

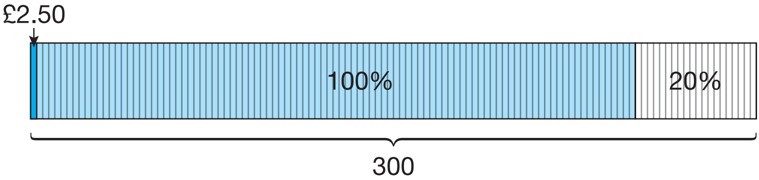
The process of dividing by 85 and multiplying by 100 can be shortcut to divide by 0.85.

When undoing a percentage increase we can again use the rod to represent what is happening.

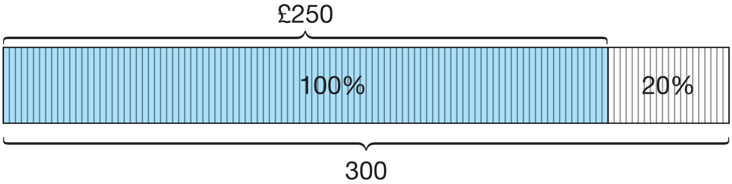
The additional 20% means that the rod is worth 120%



Dividing by 120 gives the value of 1%.



And then multiplying by 100 gives 100%, the original price.



The process of dividing by 120 and then multiplying by 100 is shortcut to dividing by 1.2.

WATCH OUT

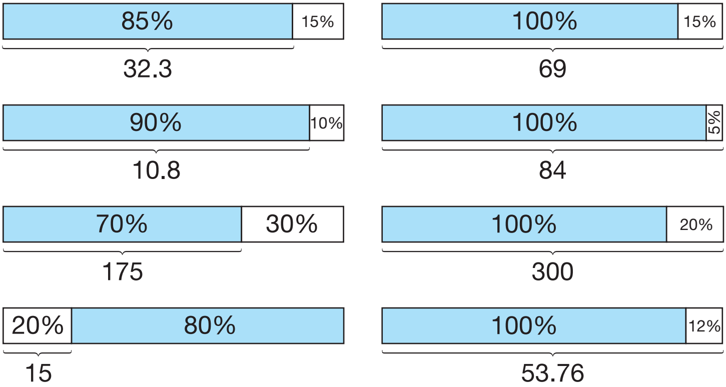
It is really tempting to try to reverse an increase of 20% by decreasing by 20%. This does not work because the amount that you are finding the percentage of is different in the two situations.

For example, increasing £100 by 20% gives £120 (increasing by 20% of 100). Decreasing £120 by 20% will take off 20% of the £120, not 20% of the original £100.

This is a very common mistake so be very careful!

TRY IT OUT

Find 100% in each of these pictures:



Find the original price for…

A shirt reduced by 15% to £32.30
A book reduced by 10% to £10.80
A TV reduced by 30% to £175
A laptop reduced by 17% to £373.50

A chocolate bar is advertised as being 15% larger than normal. It now weighs 69 grams. How much did it weigh originally?

A shop worker has a pay rise of 5% and now gets paid £8.40 per hour. How much did they get before the pay rise?

What is the problem?

Which is a better deal: 20% extra for free or 20% off the original price?

Maths outside the classroom

You will use percentages in many areas of life both in and out of school.

For example, in **Religious Studies** you might study the pillars of Islam and need to calculate 2.5% of income in order to work out how much to give as zakāt, the charitable donation expected of Muslims.

In **PE** you might need to find 70% of your maximum heart rate in order to calculate when you are improving cardio fitness.

When **baking bread** the 'baker's percentage' allows a recipe to be adapted for the quantity of flour. The amount of water needed is 60% of the weight of the flour, the amount of yeast is 5% and the amount of salt is 10%.

Answers (not in order) – percentages – page 54		
30%	80%	18.75%
3%	8%	19%
33.5%	59%	1%
95%	108%	1.25%
130%	159%	2.5%
133.5%	180%	47.5%
103.5%	75%	5%
103%	37.5%	15%
195%	95%	
3.5%	23.75%	

Answers (not in order) – percentages of amounts – page 57		
2.7	74.1	4.9
14.7	7.6	.98
43.7	1.9	93.1
45.6	36.1	0.38
39.9	19.6	38.38

Answers (not in order) – percentage changes – page 60		
Increases		
£69	£87.75	£173.25
£60.3	£68.25	£206.25
£117	£126.75	£189.75
£63	£222.75	£75
£81	£321.75	£74.75
£81.25	£165.83	£65.33
Decreases		
£51	£8.25	£3
£55.25	£156.75	£57
£48.75	£107.25	£39
£42.25	£164.18	£45
£61.75	£64.68	£140.25
£3.25	£59.70	£123.75

Answers (not in order) – reverse percentage – page 64		
38	250	£450
12	48	£250
60	75	£8
80	£38	60g
250	£12	